Math 55 Discussion problems 9 Feb

1. Show that if a, b, c, and m are integers such that $m \geq 2$, $c > 0$, and $a \equiv b \pmod{m}$, then $ac \equiv bc \pmod{mc}$.
2. Show that if n is an integer then $n^2 \equiv 0$ or $1 \pmod{4}$.
3. Show that the sum of the squares of two odd integers cannot be the square of an integer.
4. Prove that there are no solutions in integers x and y to the equation $x^2 - 5y^2 = 2$. [Hint: Consider this equation modulo 5.]
5. Show that a positive integer is divisible by 11 if and only if the difference of the sum of its decimal digits in even-numbered positions and the sum of its decimal digits in odd-numbered positions is divisible by 11.
6. How many zeros are at the end of the binary expansion of $(100_{10})!$?
7. Find the prime factorization of each of these integers.

   (a) 39 (c) 101 (e) 289

   (b) 81 (d) 143 (f) 899

8. Show that if a and b are both positive integers, then $(2^a - 1) \textbf{ mod } (2^b - 1) = 2^{a \textbf{ mod } b} - 1$.